



## Semester One Examination, 2021

### Question/Answer booklet

# MATHEMATICS SPECIALIST UNIT 1

## Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

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In words

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Your name

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### Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Number of additional  
answer booklets used  
(if applicable):

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### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet (retained from Section One)

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	92	65
<b>Total</b>					100

## Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	5	
10	5	
11	8	
12	7	
13	6	
14	8	
15	8	
16	8	
17	7	
18	8	
19	7	
20	7	
21	8	
S2 Total	92	
S2 Wt ( $\times 0.7065$ )	65%	

Section Two: Calculator-assumed

65% (92 Marks)

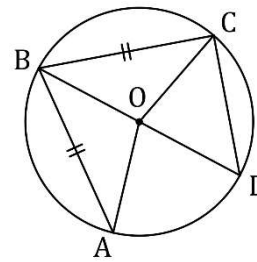
This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

**Question 9**

**(5 marks)**

Points  $A, B, C$  and  $D$  lie on the circle with centre  $O$  as shown in the diagram, where  $\angle A = 40^\circ$ ,  $AB = BC$  and  $BD$  is a diameter.



- (a) Determine the size of  $\angle AOD$ .

**(2 marks)**

- (b) Prove that  $\triangle OAD \equiv \triangle ODC$ .

**(3 marks)**

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**Question 10****(5 marks)**Determine  $\mathbf{p}$ , the vector projection of(a) a force of 320 N on a bearing  $028^\circ$  onto a force of 300 N on a bearing of  $345^\circ$ . (3 marks)(b)  $\mathbf{m}$  on  $\mathbf{n}$  where  $\mathbf{m} = (84, -13)$  and  $\mathbf{n} = (14, -48)$ . (2 marks)

## Question 11

(8 marks)

- (a) State whether each of the following statements are true or false, supporting each answer with an example or counterexample.
- (i) A quadrilateral with four congruent sides is a square. (2 marks)
- (ii) The size of one interior angle of a regular polygon with at least five sides is always obtuse. (2 marks)
- (b) Consider the statement  $\angle A \geq 90^\circ \Rightarrow \angle B < 90^\circ$  that refers to angles in triangle  $ABC$ .
- (i) Write the converse of the statement in simplest form. (1 mark)
- (ii) Write the contrapositive of the statement in simplest form. (1 mark)
- (iii) Briefly discuss the truth of the original statement, the converse statement, and the contrapositive statement. (2 marks)

## Question 12

(7 marks)

- (a) Points  $P$ ,  $Q$  and  $R$  lie on a circle of radius 5 cm, so that  $PR$  is a diameter and  $QR = 4$  cm. Determine the exact area of triangle  $PQR$ . (3 marks)

- (b) A secant meets a circle at points  $A$  and  $B$ , where  $AB = 4$  cm. A tangent to the same circle at point  $D$  intersects the secant at point  $C$ , where  $CD = 14$  cm. Given that  $BC < AC$ , determine the exact distance  $AC$  and the exact distance  $BC$ . (4 marks)

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**Question 13**

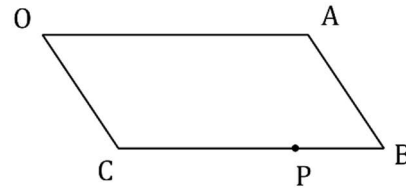
**(6 marks)**

Parallelogram  $OABC$  is shown where point  $P$  lies on side  $BC$  such that  $BP:PC = 1:3$ .

Point  $Q$ , not shown, lies on diagonal  $AC$  such that  $AQ:QC = 4:1$ .

Let  $\vec{OA} = \mathbf{a}$  and  $\vec{OC} = \mathbf{c}$ .

Express the following in terms of  $\mathbf{a}$  and  $\mathbf{c}$ .



(a)  $\vec{BO}$ . (1 mark)

(b)  $\vec{AQ}$ . (2 marks)

(c)  $\vec{QP}$ . (3 marks)

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**Question 14****(8 marks)**

A small body is acted on by force  $F_1$  of 85 N on a bearing of  $260^\circ$  and by force  $F_2$  of 45 N on a bearing of  $025^\circ$ .

(a) Sketch a diagram to show  $F_1 + F_2$  and their resultant  $R$ . (2 marks)

(b) Determine the magnitude and bearing of  $R$ . (4 marks)

(c) Express  $R$  in component form  $a\mathbf{i} + b\mathbf{j}$ . (2 marks)

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**Question 15****(8 marks)**

Consider the set of integers between 2000 and 8000 inclusive.

(a) Show that there are 462 integers in this set that are a multiple of 13. (2 marks)

(b) Determine the number of integers in this set that are  
(i) a multiple of 13 and a multiple of 18. (3 marks)

(ii) not a multiple of 13 and not a multiple of 18. (3 marks)

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**Question 16****(8 marks)**

Each letter in the word CLOUDLESS is printed individually on a card. When cards are arranged next to each other in a line, determine the number of different permutations

(a) of all the cards. (2 marks)

(b) of all the cards where all the consonants are adjacent. (2 marks)

(c) using any 4 of the cards. (4 marks)

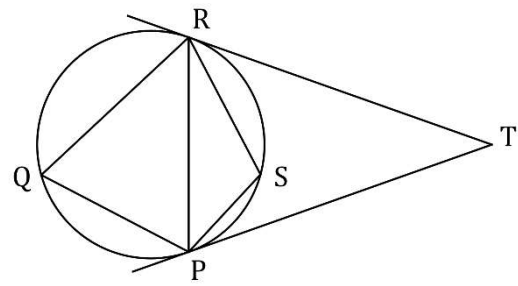
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**Question 17**

(7 marks)

- (a) The diagram shows points  $P, Q, R$  and  $S$  on the circumference of a circle. Tangents to the circle from  $P$  and  $R$  meet at point  $T$ .

Given that  $\angle T = 36^\circ$ , determine the size of  $\angle Q$  and the size of  $\angle S$ .



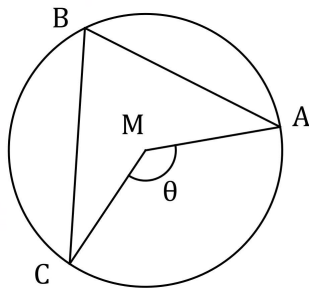
(3 marks)

- (b) In the circle shown below  $\angle A = 32^\circ$ ,  $\angle C = 24^\circ$  and  $\theta = 114^\circ$ .

Prove that  $M$  is not the centre of the circle.

(Hint: Assume that  $M$  is the centre and show that this results in a contradiction, i.e., that something “goes wrong”.)

(4 marks)



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**Question 18****(8 marks)**

Small bodies  $P$  and  $Q$  are moving with constant velocities  $(2, 0)$  m/s and  $(1, -2)$  m/s respectively.

$P$  has initial position vector  $(1, -2)$  m and  $Q$  has initial position vector  $(3, 4)$  m.

(a) Determine the distance between the bodies after five seconds. (3 marks)

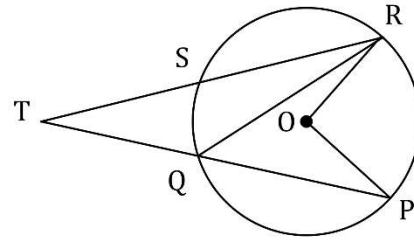
(b) Show that the distance between the bodies after  $t$  seconds is given by  $\sqrt{5t^2 - 28t + 40}$ . (3 marks)

(c) Prove that the bodies do not meet. (2 marks)

**Question 19**

(7 marks)

In the diagram shown, secants  $PQ$  and  $RS$  intersect at  $T$ , a point outside the circle with centre  $O$ .



- (a) Determine the size of  $\angle RQP$  and  $\angle ROP$  when  $\angle T = 36^\circ$  and  $\angle SRQ = 19^\circ$ . (2 marks)

- (b) Prove that when secants  $PQ$  and  $RS$  intersect at  $T$ , a point outside the circle with centre  $O$ , then  $\angle T = \frac{1}{2}(\angle ROP - \angle SOQ)$ . (4 marks)

- (c) Determine the size of  $\angle T$  when  $\angle SOQ = 42^\circ$  and  $\angle ROP = 82^\circ$ . (1 mark)

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**Question 20****(7 marks)**

- (a) A manufacturer makes the same plastic toy figure in nine different colours and sells them in packs of four. The toys inside each pack are randomly chosen from the production line in such a way that all are of a different colour.

Determine the least number of packs that a retailer should buy from the manufacturer to be certain of obtaining at least five packs containing the same colour combination of toys.  
(3 marks)

- (b) A set of cards is numbered with all the integers from 1 to 18 inclusive. The cards are shuffled, placed face down and then the cards turned over one by one.

Determine how many cards must be turned over to be certain that at least one of the numbers on a face up card will be exactly half of the number on another face up card.  
(4 marks)

**Question 21****(8 marks)**

Harbour  $Y$  lies on a bearing of  $065^\circ$  from harbour  $X$  and the straight line distance between the harbours is 43 km. Between the harbours, a steady current is moving in a south easterly direction at a speed of 1.5 metres per second.

A boat with a cruising speed of 5.5 metres per second is to travel from harbour  $X$  to harbour  $Y$  in the least possible time.

- (a) Sketch a diagram, roughly to scale, to show the resultant of the sum of the vectors for the boat and the current. (2 marks)

- (b) Determine the bearing the boat should steer, to the nearest degree, and the time its journey takes, to the nearest minute. (6 marks)

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Supplementary page

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